MAST90079 AMSI Summer School

Credit Points:	12.5
Level:	9 (Graduate/Postgraduate)
Dates & Locations:	2016, Parkville This subject commences in the following study period/s: Summer Term, Parkville - Taught on campus.
Time Commitment:	Contact Hours: 28 hours over 4 weeks Total Time Commitment: 170 hours
Prerequisites:	Subject coordinator approval, upon endorsement by your current supervisor and course coordinator, is required upon application.
Corequisites:	None
Recommended Background Knowledge:	None
Non Allowed Subjects:	None
Core Participation Requirements:	For the purposes of considering request for Reasonable Adjustments under the Disability Standards for Education (Cwth 2005), and Student Support and Engagement Policy, academic requirements for this subject are articulated in the Subject Overview, Learning Outcomes, Assessment and Generic Skills sections of this entry. It is University policy to take all reasonable steps to minimise the impact of disability upon academic study, and reasonable adjustments will be made to enhance a student's participation in the University's programs. Students who feel their disability may impact on meeting the requirements of this subject are encouraged to discuss this matter with a Faculty Student Adviser and Student Equity and Disability Support: http://services.unimelb.edu.au/disability
Coordinator:	Assoc Prof Sanming Zhou
Contact:	Email: **sanming@unimelb.edu.au (mailto:sanming@unimelb.edu.au)**
Subject Overview:	The AMSI Summer School is a four-week program hosted at RMIT by the Australian Mathematical Sciences Institute. This subject will give students the opportunity to attend subjects taught by eminent lecturers from around Australia, creating an opportunity to study areas of mathematical sciences and cognate disciplines that may not be otherwise available at the University of Melbourne. AMSI Summer School is an exciting opportunity for mathematical sciences students from around Australia to come together over the summer break to develop their skills and networks. Students can choose from a selection of available modules which are individually detailed below. No course within the AMSI Summer School program that substantially covers material available in existing University of Melbourne postgraduate Mathematics and Statistics subjects will be available. The Schoolof Mathematics and Statistics determines the subset of allowed modules that students can choose from. This subject is only available to students enrolled in the Master of Science (Mathematics and Statistics) or the Graduate Diploma in Science (Advanced) in the Mathematics and Statistics stream. - **Available Modules** *Calculus of variations: Theory and Practice* In many physical problems, the solutions we seek minimise an energy. As a consequence, these solutions will also satisfy a partial differential equation. For example, minimisers of the Dirichlet energy satisfy the Laplace equation; in another example, surfaces that minimise area satisfy the minimal surface equation. These problems have beautiful geometric, analytic and practical aspects.

In this course you will see how to find the partial differential equation associated with an energy; how to ensure that minimisers exist; how to deal with constraints; how to model these problems numerically; and the techniques of stability analysis. Topics covered will include:

- Energy functionals
- The first variation + Euler Lagrange equation
- Existence
- Constraints
- Regularity (possibly)
- Eigenvalues (possibly)

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Conic Programming

Conic programming is one of the core areas in modern optimisation. Conic models such as semidefinite programming have broad applications in real life, but at the same time present nontrivial computational challenges that result in rich underlying theory and interesting research problems. The aim of these lectures is to go over the fundamentals of conic optimisation and give an overview of the research field. This includes the basics of structured convex analysis, modern optimisation methods, modelling and relaxation techniques, complexity and ill-posedness issues, and a discourse on current research directions and open problems. Topics covered will include:

- Convex sets: convex and conic hulls, relative interior, recession directions, extreme points and facial structure, polyhedral convex sets.
- Duality and separation: projection operator, separation theorem, polars, dual cone, ill-posed problems and distance to infeasibility.
- Semidefinite programming (SDP) and generalisations: the cone of positive semidefinite matrices, spectrahedra, strong duality and facial reduction, copositive programming, hyperbolicity cones.
- Algorithms and complexity: optimisation problems and feasibility problems, condition numbers and complexity of classic algorithms.

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Linear Control Theory and Structured Markov Chains

This course covers the core elements of linear dynamical systems, control theory and Markov chains. The field of linear dynamical systems is all about deterministic mathematical models utilizing linear differential equations. Linear control theory considers the case where the dynamical systems are modified by means of a feedback mechanism. Here system behaviour is engineered to be governed by actuators and sensed by sensors. The general constructs are system models, feedback control, observers and optimal control laws under quadratic costs. The mathematics needed to master basic linear control theory is mostly centred around linear algebra and integral transforms. Moving onto inherently stochastic models arising in some engineered and biological systems, one considers Markov chains with discrete state spaces. Such Markov chains are naturally related to linear control theory since the state transition probabilities of Markov chains evolve as a linear dynamical system.Other relationships between these objects also exist. This merits studying linear dynamical systems, control theory and Markov chains simultaneously. In this course the students will explore these related fields together with applications, computation and theory. Topics covered will include:

- Motivating examples from science and engineering
- Linear Time Invariant Systems and Probability Distributions
- Linear Dynamical Systems and Markov Chains
- Selected Markov Models
- State feedback, observers and separation in their design
- Lyapounov Stability for both deterministic and stochastic systems
- Bellman stye optimal control for both deterministic and stochastic systems

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Modern Numerical Methods for Diffusion Equations on Generic Grids

Porous media flows models, such as those involved in oil recovery or carbon storage, are too complex to be analytically solved. Numerical approximation is the only way to obtain quantitative and qualitative information on the solutions to these models. In field applications, several engineering constraints exist that must be taken into account in the mathematical study of

	the models and of their approximation: discontinuous permeability, unstructured grids with degenerate cells, etc. This course covers numerical methods — mostly finite volume schemes — developed in the past 15 years to tackle the numerical approximation of diffusion models under these engineering constraints. The construction of these schemes hinges on the analytical properties of the equations. We will therefore first detail these properties, before presenting the numerical schemes. Elements of the convergence analysis of the schemes, under constraints compatible with practical applications, will be given; we will also tackle questions pertaining to the implementation of the methods. For the sake of accessibility of the course, we will only consider linear steady diffusion; however, the material covered in this course also forms the core of the techniques required to deal with non-linear and transient real-world models. Topics covered will include: # Weak formulation of linear diffusion equations, properties of solutions # Conforming Galerkin methods # Non-conforming Finite Element methods # Elements of convergence analysis: gradient scheme framework # Hybrid Mimetic Mixed methods # Finite volume methods on orthogonal grids - *Projective Geometry* Cayley famously said that “projective geometry is all geometry” since it has the weakest formalisation of all the interesting geometries (i.e., those with large symmetry groups) and we can easily recover the Euclidean and affine geometries by adopting extra structure. There are essentially four school’s of practice in geometry: axiomatic, synthetic, analytic and transformational. In most of the modern day secondary and tertiary education in geometry, the analytic perspective holds sway. This course redresses this situation by emphasising the synthetic and transformational approach to projective geometry. The synthetic approach recaptures what geometry is all about, whilst we bring in the modern transformational perspective initiated by Hjelmslev, Hessenberg, Thomsen and Bachmann. This course will also be an introduction to reflection groups and permutation group theory, with continual interplay with the classical geometries that we have seen along the way. Topics covered will include: # The real Euclidean plane, isometries and similarities # The real affine plane and affine transformations # The real projective plane and its collineation group - Further information on AMSI can be found at there website - **http://ss16.amsi.org.au/program/ (http://ss16.amsi.org.au/program/)** Students may gain credit for this subject only once.
Learning Outcomes:	After completing this subject students should: # have gained an understanding at postgraduate level of the mathematics and statistics of the selected topic; # be familiar with the basic mathematical techniques used in the area of the selected topic; # appreciate the role of the newly learned mathematical results in the broader context of mathematical sciences and their potential applications to solving real world problems; # have the ability to pursue further studies in the area of the selected topic and related areas.
Assessment:	Calculus of variations: Theory and Practice 3 written assignments totalling up to 50 pages (45%: three assignments worth 15% each, due Monday 11th January, Wednesday 20th January and Friday 5th February (to be emailed)), 1 small group project up to 50 pages (55%: an extended investigation of a particular variational problem working in small groups of 2-4, with a presentation on Thursday 28th January and Friday 29th January - Conic Programming 2 written assignments totalling up to 50 pages (50%: two assignments each worth 25% due in the third and fourth weeks respectively), and 1 three hour written examination (worth 50%) of up to 50 pages at home institutions in the first week in February. - Linear Control Theory and Structured Markov Chains up to 30 pages of written assignment due in the second, third and fourth weeks respectively (35%: average of top two grades out of three assignments (each

	assignment is budgeted as requiring 10-20 hours of work)), in-class quizzes (15%: average of top five grades out of seven in-class short-quizzes (10 min each)), and a 36-hour take home written examination in the first week in February (worth 50%) - Modern Numerical Methods for Diffusion Equations on Generic Grids up to 10 pages of written assignment due in the second and third weeks (30%: two assignments worth 10% and 20% respectively), up to 5 hours of coding (computer assignment worth 20%) in the third week, and a 3 hour written examination (worth 50%) in the fourth week - Projective Geometry 2 written assignments of up to 50 pages combined due in the third and fourth weeks (40%: two assignments worth 20% each), and 1 written examination of up to 3 hour (worth 60%) in the fourth week - Assessment will be held during the 4 weeks of the summer school with exam components within 2 weeks of the end of the summer school.
Prescribed Texts:	Details of prescibed text will be provided after enrolment.
Breadth Options:	This subject is not available as a breadth subject.
Fees Information:	Subject EFTSL, Level, Discipline & Census Date, http://enrolment.unimelb.edu.au/fees
Generic Skills:	In addition to learning specific skills that will assist students in their future careers in science, they will have the opportunity to develop generic skills that will assist them in any future career path. These include: # problem-solving skills: the ability to engage with unfamiliar problems and identify relevant solution strategies; # analytical skills: the ability to construct and express logical arguments and to work in abstract or general terms to increase the clarity and efficiency of analysis; # collaborative skills: the ability to work in a team; # time-management skills: the ability to meet regular deadlines while balancing competing commitments.